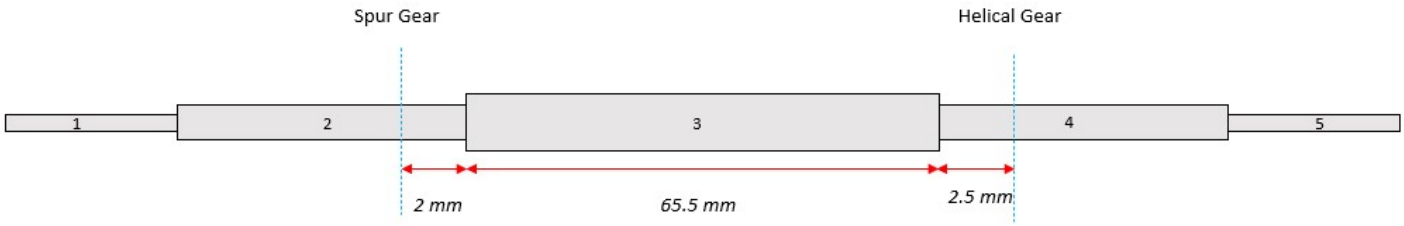
Problem: Verify that the angle of twist between the gears is less than 3 degrees/m

Known Parameters:

$E := 200\ \text{GPa}$

$G := 77\ \text{GPa}$

$T := 1000\ \text{N m}$



Section	1	2	3	4	5
d (mm)	35	45	Stock size (57.15)	50	35
I (10^{-8} m^4)	7.37	20.1	52.4	30.7	7.37
J (10^{-8} m^4)	14.7	40.3	105	61.4	14.7
Start position (x_i mm)	0	10.5	82	147.5	229.5
End position (x_f mm)	10.5	82	147.5	229.5	240
Segment length (m)	0.0105	0.0715	0.0655	0.082	0.0105

Assumptions:

1. The beam is made of a homogeneous material (has a constant modulus of elasticity across its length).
2. The material is in the linear elastic region (Hooke's Law is applicable).
3. Deformed angels and displacements are small.
4. The longitudinal axis, which acts within the neutral surface, does not experience any change in length.
5. All cross-sections of the beam remain plane and perpendicular to the longitudinal axis during deformation.

Angle of Twist:

We can use the equation below to solve for the angle of twist between bth gears:

$$\phi := \frac{T}{G} \cdot \Sigma \left(\frac{L}{J} \right)$$

$$\phi := \frac{T}{G} \cdot \left(\frac{L_1}{J_1} + \frac{L_2}{J_2} + \frac{L_3}{J_3} \right) \quad (1)$$

From the known parameters above, we can find the values of length and polar moments of inertia for each section:

$L_1 := 0.002\ \text{m}$ $\quad J_1 := 0.000000403\ \text{m}^4$

$L_2 := 0.0655\ \text{m}$ $\quad J_2 := 0.00000105\ \text{m}^4$

$L_3 := 0.0025\ \text{m}$ $\quad J_3 := 0.000000614\ \text{m}^4$

Substituting values into equation (1) :

$$\phi := \frac{T}{G} \cdot \left(\frac{L_1}{J_1} + \frac{L_2}{J_2} + \frac{L_3}{J_3} \right)$$

$$\phi = 0.00092747 \text{ rad}$$

$$\phi = 0.05314027 \text{ deg}$$

We can use the following equation to find the rotation per meter: :

$$L := L_1 + L_2 + L_3$$

$$\frac{\phi}{L} = 0.75914668 \frac{\text{deg}}{\text{m}}$$

Conclusion :

Since 0.759 deg/m is less than 3 deg/m, the **anglular twist is acceptable** for this stepped shaft.